

# RADIATIVE ENERGY TRANSFER MODEL FOR PRE-STRESSED ORTHOTROPIC PLATE ON VISCO-ELASTIC FOUNDATION

Qiang Zhong

*CAS Key Laboratory of Mechanical Behavior and Design of Materials, Department of Modern Mechanics, University of Science and Technology of China, Hefei, Anhui 230026, PR China*  
email: qzhong@mail.ustc.edu.cn

Haibo Chen

*CAS Key Laboratory of Mechanical Behavior and Design of Materials, Department of Modern Mechanics, University of Science and Technology of China, Hefei, Anhui 230026, PR China*  
email: hbchen@ustc.edu.cn

Alain Le Bot

*LTDS-UMR 5513 CNRS-Ecole Centrale de Lyon 36, Avenue Guy de Collongue 69134 Ecully, France*  
email: alain.le-bot@ec-lyon.fr

In this study, we develop the radiative energy transfer method for pre-stressed orthotropic plate vibrational on visco-elastic foundation. For orthotropic plate system, the formula of the energy transfer depends not only on the distance between the observation point and source point, but also on the head angle of the wave. And the in-plane pre-stresses and visco-elastic foundation applied on the orthotropic plate can change the wave number, group velocity and input power. Some numerical simulations are performed for a typical tyre model to verify its validation. The simulation results, such as energy density distribution and energy flow field, are compared with the traditional modal superposition solutions for various frequencies and damping loss factors. We analyze the influence of elastic stiffness and pre-stresses on the dynamic response of the orthotropic plate by the proposed model.

Keywords: radiative energy transfer method, orthotropic plate, in-plane pre-stresses, visco-elastic foundation

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## 1. Introduction

Methods related to high frequency wave propagation have been introduced into response analysis of structures in recent decades. Statistical energy analysis (SEA) represents a most popular method for analyzing the high-frequency response of built-up structures under mechanical or acoustical load [1]. Power flow analysis (PFA) or Energy finite element method (EFEM) offer an alternative method to SEA which does not present the detail local energy distribution in a subsystem[2, 3], but their effectiveness only exists in the high frequency vibration analysis of the one-dimensional system[4].

In order to simulate the energy flow in a high frequency vibrational orthotropic medium, D.-H. Prak and S.-Y. Hong[5] established a model by developing the PFA from isotropic medium to orthotropic

medium. Based on the study of D.-H. Prak and S.-Y. Hong, X. Y. Yan[6] proposed the EFEM models of orthotropic and anisotropic plates to predict the highfrequency vibration response of composite structures. Later, Miaoxia Xie[8] developed the EFEM model for thin orthotropic plate to analyze the high frequency vibration of a rectangular stiffened thin orthotropic plate. Noted that all these derivation results show that the power flow intensity is proportional to the gradient of the energy density in the orthotropic medium due to the reverberant wave field assumption, and it is consistent with the Fourier’s law of heat conduction. However, Langley[9] and Aain Le Bot reveal[4] that the high frequency vibration response for a two-dimensional system is determined by the direct wave field, that is to say, EFEM has a large error in estimating the response of a two-dimensional system.

In general, the vibrational characteristics of orthotropic plates have been widely investigated from low frequency to high frequency. However, the existing energy flow methods for high frequency vibrational orthotropic plate can not give a reliable local energy density response. Thus, the aim of this study is developing a RETM for pre-stressed orthotropic plate vibrating on visco-elastic foundation.

## 2. Basic theory of radiative energy transfer method

### 2.1 Statistical parameters

The governing equation for out-of-plane vibrations in a prestressed orthotropic plate resting on an elastic foundation excited by a harmonic point force located at  $(x_0, y_0)$  is[10]

$$\left[ \left( \sqrt{D_x} \frac{\partial^2}{\partial x^2} + \sqrt{D_y} \frac{\partial^2}{\partial y^2} \right)^2 - \left( T_x \frac{\partial^2}{\partial x^2} + T_y \frac{\partial^2}{\partial y^2} \right) + c \frac{\partial}{\partial t} + k + m \frac{\partial^2}{\partial t^2} \right] w = F e^{i\omega t} \delta(x-x_0, y-y_0) \quad (1)$$

where  $w(x, y, t)$  is the transverse displacement and  $m$  is the mass per unit area of the plate.  $D_x$  and  $D_y$  are the bending stiffness in  $x$ - and  $y$ -directions,  $T_x$  is the  $x$ -component of the tensile force,  $c$  is the viscous damping coefficient,  $k$  is the stiffness per unit area of the elastic foundation,  $F$  and  $\omega$  are the amplitude and the circular frequency of excitation.

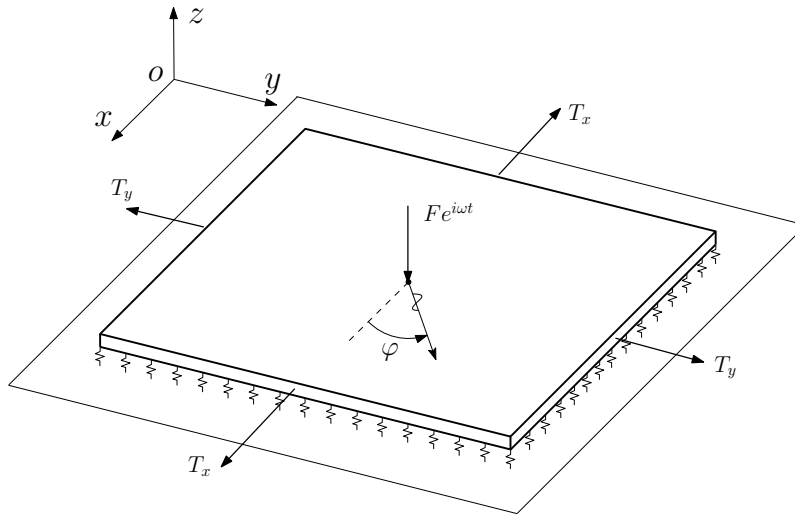


Figure 1: A prestressed rectangular orthotropic plate.

By substituting a travelling wave solution and considering the undamped case ( $c = 0$ ), we get the

dispersion equation.

$$(\sqrt{D_x} \cos^2 \varphi + \sqrt{D_y} \sin^2 \varphi)^2 \kappa^4 + (T_x \cos^2 \varphi + T_y \sin^2 \varphi) \kappa^2 + k - m\omega^2 = 0 \quad (2)$$

Phase velocity is defined as

$$c_p(\varphi) = \frac{\omega}{\kappa} \quad (3)$$

and group velocity is defined as

$$c_g(\varphi) = \frac{\partial \omega}{\partial \kappa} \quad (4)$$

The attenuation coefficient is defined as

$$m(\varphi) = \frac{\eta \omega}{c_g(\varphi)} \quad (5)$$

where  $\eta$  is the structural damping loss factor.

Eqs. (3), (4) and (5) are wave velocities and attenuation coefficient for the pre-stressed orthotropic plate vibrating on elastic foundation. The influence of elastic stiffness of the foundation and in-plane tension forces on the three variables are involved. And for orthotropic medium, they are functions of the direction angle of propagation wave is significant.

## 2.2 Radiative energy transfer method

The radiative energy transfer method is a high frequency theory of sound and vibration. This theory belongs to geometrical acoustics and therefore its validity is limited to short wavelengths (i.e. high frequencies). As such, the method neglects interference effects and only provides mean values of sound levels when sources are random. The theory is inspired from the so-called "view factor method" in thermics. The primary variables of the theory are energy density and energy flux in the domain (vibrating structure or acoustic room). The theory is based on a local energy balance on the boundary and aims to calculate the power reflected from the boundary. This leads to a boundary integral equation on the reflected power in the form of Fredholm equation of the second type, as shown in Eqs. (1) and (2). This may be solved by any standard algorithm such as the collocation or Galerkin method.

$$W(M) = \int_{\Omega} \rho(S) G(S, M) d\Omega + \int_{\Gamma} \sigma(P, M) G(P, M) d\Gamma \quad (6)$$

$$\mathbf{I}(M) = \int_{\Omega} \rho(S) \mathbf{H}(S, M) d\Omega + \int_{\Gamma} \sigma(P, M) \mathbf{H}(P, M) d\Gamma \quad (7)$$

This method is basically equivalent to the ray-tracing method but requires a quite different implementation procedure. Among the advantages, it is possible to apply it to the vibration analysis of plates or shells even if they are coupled and their number is very large.

For weak anisotropic plate, we assume that the group velocity and phase velocity are in same direction. Rays propagating in different directions keep straight line, but the group velocity depends on the direction angle.

## 3. Discussion

For the values of physic parameters of the rectangular orthotropic plate, we can find in reference[10] for a tyre model as shown in Table. 1. The first example is given to show the difference of the wave fields between isotropic plate and orthotropic plate. When the stiffness of the two principal axes of the

Table 1: Parameter numerical values of the orthotropic plate.

Parameter	$a$	$b$	$M$	$D_x$	$D_y$	$\nu_x$	$\nu_y$
Units	$m$	$m$	$kg/m^2$	$N \cdot m$	$N \cdot m$	-	-
Value	1	1	12.4	20	8	0.4	0.16

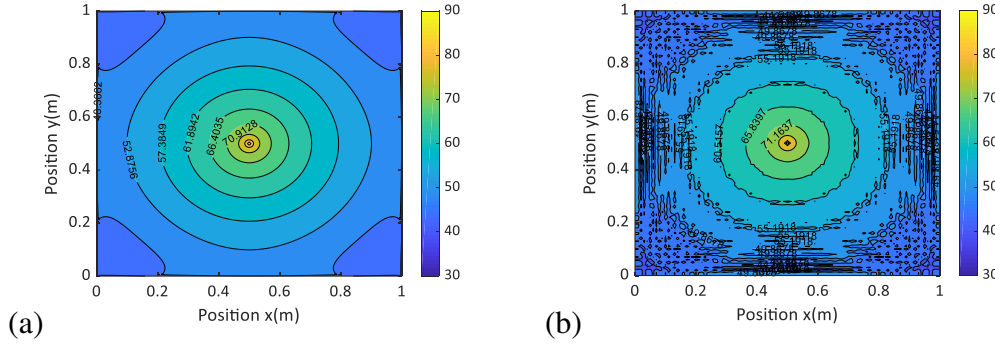


Figure 2: Energy distribution of isotropic plate  $D_y = D_x$ ,  $f = 5000Hz$ ,  $\eta = 0.1$ , (reference= $10^{-12} J/m^2$ ): (a) RETM; (b) Exact solution.

orthotropic plate is equal  $D_y = D_x$ ,  $f = 5000Hz$ ,  $\eta = 0.1$ , the energy density distribution near the excitation point is more like a cylindrical as shown in Fig 2. Comparing 2(a) and 2(b), it can be seen that RETM is effective for estimating the energy distribution of isotropic plate.

With the proposed model in this study, the energy density distribution of orthotropic plate is calculated as shown in Fig 3. Since the bending stiffness of x-direction is greater than y-direction, the wave speeds in two direction exists  $c_{gx} > c_{gy}$ , the dissipation of energy in x-direction is less than y-direction  $m_x < m_y$  at frequency  $\omega$ . The energy distribution of orthotropic plate is not like isotropic plate, the energy density contour line of the orthotropic plate near the excitation point is non-cylindrical.

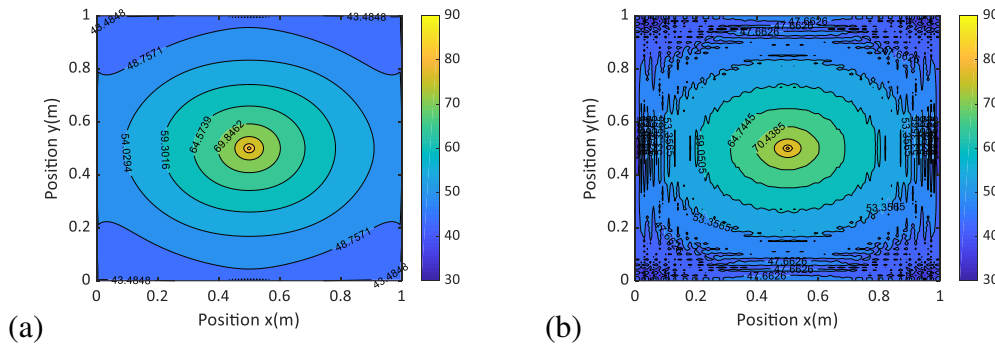


Figure 3: Energy distribution of orthotropic plate,  $f = 5000Hz$ ,  $\eta = 0.1$ : (a) RETM; (b) Exact solution.

To further illustrate the accuracy of the model presented in this study, the energy responses at central lines (x- and y-direction) predicted by RETM are compared with the exact modal superposition solution. And to show the difference between the existing energy model and RETM for the orthotropic plate system, the results by RETM are also compared with the PFA solution[5] as shown in Fig 4.

Using the parameters of the previous example, we calculated the energy distribution at  $f = 2500Hz$  and  $\eta = 0.05$  as shown in Fig 5. It can be seen that even with the application of in-plane pre-stress and elastic support, the response estimation model provided in this paper still has a good estimation result.

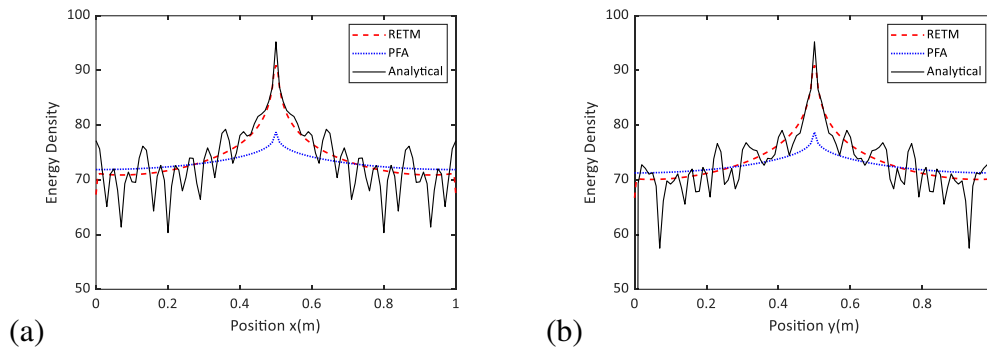


Figure 4: Energy distribution at central line,  $f = 1000\text{Hz}$ ,  $\eta = 0.05$ : (a) x-direction; (b) y-direction.

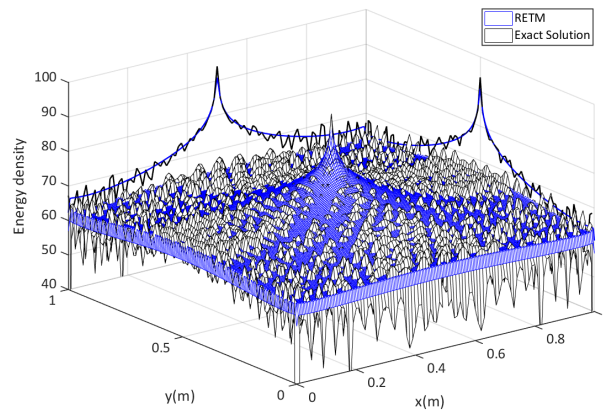


Figure 5: Energy density distribution with both prestresses and elastic foundation effect,  $T_x = 3 \times 10^4\text{N/m}$ ,  $T_y = 8 \times 10^4\text{N/m}$ ,  $k_w = 1.3 \times 10^6\text{N/m}^3$ ,  $f = 2500\text{Hz}$ ,  $\eta = 0.05$ .

## 4. Conclusion

This study developed a radiative energy transfer method for pre-stressed orthotropic plate vibrating on visco-elastic foundation. Firstly, the present model gives a more accurate solution than the existing PFA solution for vibrational orthotropic plate under a high frequency harmonic point load. Secondly, the shape of wavefront for orthotropic plate is no longer cylindrical, it is elliptical.

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